Quantum computing and quantum communication

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UNI BASEL

What will we learn ?

- elements of quantum information
  - qubits
  - superposition and entanglement
  - 1- and 2-qubit gates
  - no-cloning theorem
  - Deutsch algorithm
- error correction, encryption, teleportation
- “hardware” for quantum computers

references:

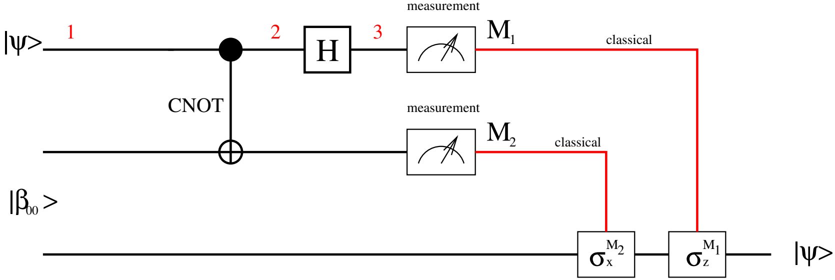
N.D. Mermin, Quantum computer science, Cambridge University Press

M.A. Nielsen and I.L. Chuang, Quantum computation and quantum information, Cambridge University Press

Lecture notes by C. Bruder

Quantum teleportation

- Cloning a quantum state is impossible (no cloning theorem)
- However, it is possible to teleport it even if only classical signals can be transmitted
- If Alice and Bob have one half each of $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- It is possible for Alice to transmit an unknown state $|\psi\rangle = a|0\rangle + b|1\rangle$ to Bob, using only classical information



- Top two lines represent Alice's system, the last one Bob's
- 1: $|\psi\rangle|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[a|0\rangle(|00\rangle + |11\rangle) + b|1\rangle(|00\rangle + |11\rangle)]$
- 2: $\frac{1}{\sqrt{2}}[a|0\rangle(|00\rangle + |11\rangle) + b|1\rangle(|10\rangle + |01\rangle)]$
- 3: $\frac{1}{2}[a(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + b(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)]$
- $=\frac{1}{2}[|00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a|1\rangle + b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)]$

Quantum teleportation II

- If Alice measures 00, Bob's system will be in state $|\psi\rangle$
- If she measures something else and tells him (=classical communication), Bob can "fix it" such that his state is equal to $|\psi\rangle$

Bell Test

- Many versions of Bells's test exist
- Clauser-Horne-Shimony-Holt(CHSH)
- Suppose Alice and Bob perform independent measurements
- These measurements can be performed in two settings A_1, A_2 for Alice, and B_1, B_2 for Bob
- Independent measurements would be possible if the two qubit state is factorizable
- All measurements can have possible outcome $\{A_1, A_2, B_1, B_2\} \in \{+1, -1\}$
- ± 1 could represent $\{\uparrow, \downarrow\}$ electron spin, or $\{H, V\}$ photon polarization

Bell Test II

- Measuring spin along $\hat{z}$ (corresponds to $\hat{\sigma}_z$ gives us $\{+1, -1\}$)
- Measuring spin along $\hat{x}$ (corresponds to $\hat{\sigma}_x$ gives us $\{+1, -1\}$)
- Measuring spin along $\hat{y}$ (corresponds to $\hat{\sigma}_y$ gives us $\{+1, -1\}$)
- Consider $\mathcal{C} \equiv (A_1 + A_2)B_1 + (A_1 - A_2)B_2$
- Either $A_1 + A_2$ is 0 in that case $A_1 - A_2$ is ± 2
- Or $A_1 - A_2$ is 0 in that case $A_1 + A_2$ is ± 2
- $\Rightarrow \mathcal{C} = \pm 2$

Bell Test III

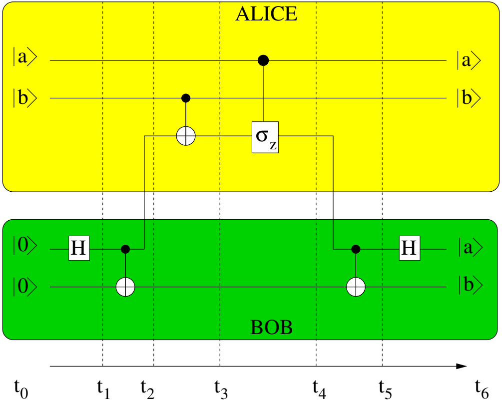
- Measuring spin along a general direction $\hat{n}$ given by azimuthal angle θ and polar angle ϕ corresponds to $\hat{\sigma}_{\theta,\phi}$
- $\hat{\sigma}_{\theta,\phi} = \cos(\phi)\sin(\theta)\hat{\sigma}_x + \sin(\phi)\sin(\theta)\hat{\sigma}_y + \cos(\theta)\hat{\sigma}_z$
- For simplicity let Alice and Bob measure spin along a general direction in the XZ plane ($\phi = 0$)
- A typical measurements operator is $\hat{\sigma}_{\alpha,0} \otimes \hat{\sigma}_{\beta,0}$
- $\begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & -\cos(\alpha) \end{bmatrix} \otimes \begin{bmatrix} \cos(\beta) & \sin(\beta) \\ \sin(\beta) & -\cos(\beta) \end{bmatrix}$
- Bell State: $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Bell Test IV

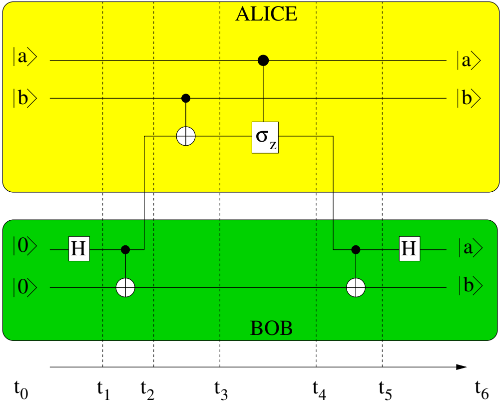
- $\langle \beta_{00} | \hat{\sigma}_{\alpha,0} \otimes \hat{\sigma}_{\beta,0} | \beta_{00} \rangle = \cos(\alpha - \beta)$
- $|\langle \mathcal{C} \rangle| \leq \langle |\mathcal{C}| \rangle = 2$
- $|\langle \mathcal{C} \rangle| = |\langle A_1 B_1 \rangle + \langle A_2 B_1 \rangle + \langle A_1 B_2 \rangle - \langle A_2 B_2 \rangle|$
- Choose angles $\alpha_1 = 0$, $\alpha_2 = \frac{\pi}{2}$, $\beta_1 = \frac{\pi}{4}$, and $\beta_2 = -\frac{\pi}{4}$
- $|\langle \mathcal{C} \rangle| =$
  $|\cos(\alpha_1 - \beta_1) + \cos(\alpha_2 - \beta_1) + \cos(\alpha_1 - \beta_2) - \cos(\alpha_2 - \beta_2)|$
- $|\langle \mathcal{C} \rangle| = 2\sqrt{2}$ violates CHSH version of Bell inequality
- Violation of the inequality $|\langle \mathcal{C} \rangle| \leq 2$ demonstrates entanglement

Superdense Coding: preliminaries

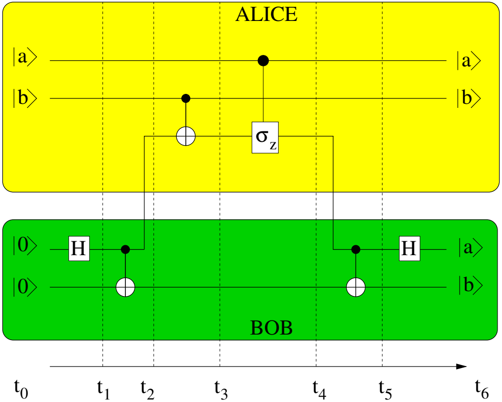
- Reminder: $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- $|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$
- $|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
- $|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
- $\hat{\sigma}_x \otimes I|\beta_{00}\rangle = |\beta_{01}\rangle$
- $\hat{\sigma}_z \otimes I|\beta_{00}\rangle = |\beta_{10}\rangle$
- $\hat{\sigma}_y \otimes I|\beta_{00}\rangle = |\beta_{11}\rangle$ (up to a phase)
- Consider that Alice and Bob share a Bell state $|\beta_{ij}\rangle$
- Alice can convert this Bell state into any other Bell state herself (with no help from Bob)



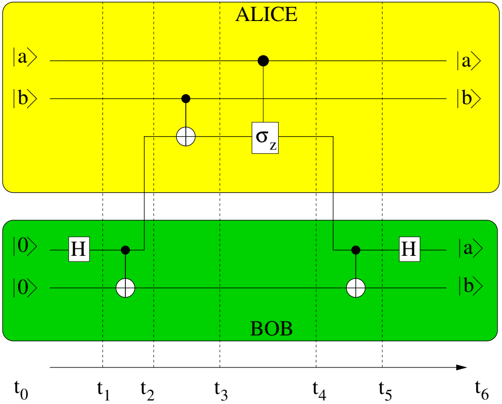
- State at t_0 is $|a\rangle|b\rangle|0\rangle|0\rangle$
- State at t_1 is $|a\rangle|b\rangle(\frac{|0\rangle+|1\rangle}{\sqrt{2}})|0\rangle$
- State at t_2 is $|a\rangle|b\rangle(\frac{|00\rangle+|11\rangle}{\sqrt{2}})$



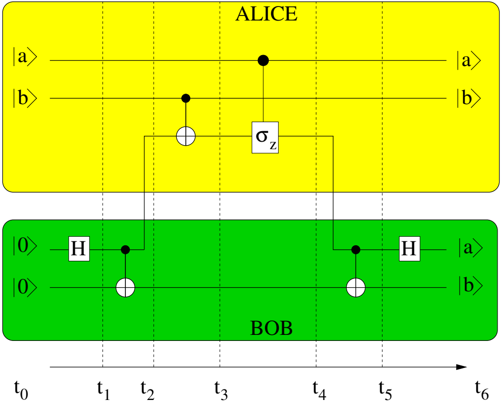
- State at t_2 is $|a\rangle|b\rangle(\frac{|00\rangle+|11\rangle}{\sqrt{2}})$
- State at t_3 is $|0\rangle(\frac{|000\rangle+|011\rangle}{\sqrt{2}})$ or $|0\rangle(\frac{|110\rangle+|101\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|000\rangle+|011\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|110\rangle+|101\rangle}{\sqrt{2}})$



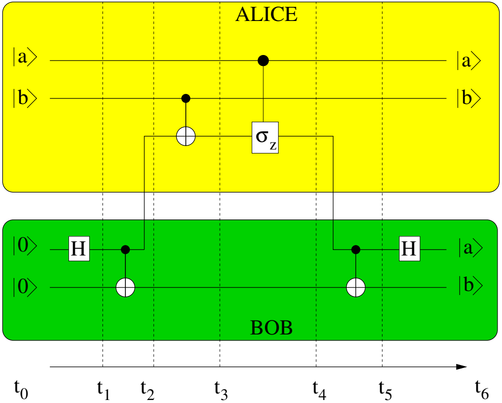
- State at t_3 is $|0\rangle(\frac{|000\rangle+|011\rangle}{\sqrt{2}})$ or $|0\rangle(\frac{|110\rangle+|101\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|000\rangle+|011\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|110\rangle+|101\rangle}{\sqrt{2}})$
- State at t_4 is $|0\rangle(\frac{|000\rangle+|011\rangle}{\sqrt{2}})$ or $|0\rangle(\frac{|110\rangle+|101\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|000\rangle-|011\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{-|110\rangle+|101\rangle}{\sqrt{2}})$



- State at t_4 is $|0\rangle(\frac{|000\rangle+|011\rangle}{\sqrt{2}})$ or $|0\rangle(\frac{|110\rangle+|101\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|000\rangle-|011\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{-|110\rangle+|101\rangle}{\sqrt{2}})$
- State at t_5 is $|0\rangle(\frac{|000\rangle+|010\rangle}{\sqrt{2}})$ or $|0\rangle(\frac{|111\rangle+|101\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{|000\rangle-|010\rangle}{\sqrt{2}})$ or $|1\rangle(\frac{-|111\rangle+|101\rangle}{\sqrt{2}})$



- State at t_5 is $|0\rangle|0\rangle(\frac{|00\rangle+|10\rangle}{\sqrt{2}})$ or $|0\rangle|1\rangle(\frac{|11\rangle+|01\rangle}{\sqrt{2}})$ or $|1\rangle|0\rangle(\frac{|00\rangle-|10\rangle}{\sqrt{2}})$ or $|1\rangle|1\rangle(\frac{-|11\rangle+|01\rangle}{\sqrt{2}})$
- State at t_6 is $|0\rangle|0\rangle|0\rangle|0\rangle$ or $|0\rangle|1\rangle|0\rangle|1\rangle$ or $|1\rangle|0\rangle|1\rangle|0\rangle$ or $|1\rangle|1\rangle|1\rangle|1\rangle$

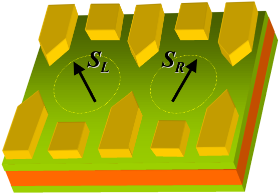


- Single qubit can carry two bits of classical information
- No information about *a* and *b* in one qubit alone, only the combined system of Alice and Bob carries the information

physical implementations of quantum bits

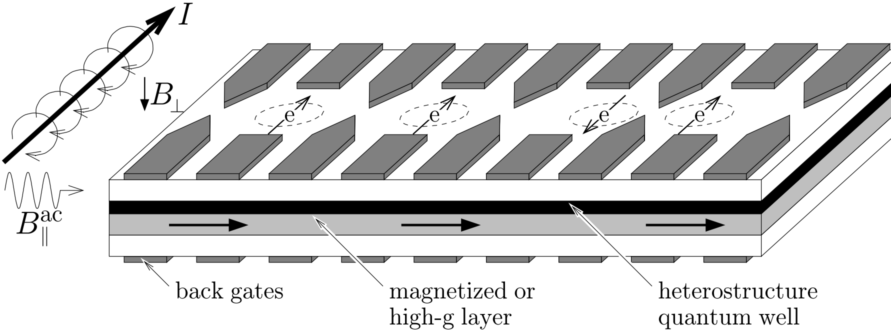
- Ions in magnetic traps Cirac and Zoller
- Spins in large molecules + NMR Cory et al., Gershenfeld and Chuang
- Atoms in optical lattices Jaksch et al.
- Electron spins in semiconductor quantum dots Loss and DiVincenzo
- Superconducting flux qubit Mooij et al., Ioffe et al.
- Superconducting charge qubit Schön et al., Averin
- Superconducting phase qubit Martinis et al.

Electron spins in quantum dots



- A two-dimensional electron gas (2DEG) can be realized in semiconductor heterostructures
- 2DEG can be structured by gate electrodes (negative potential repels electron gas under the electrode)
- quantum dots may be formed which contain a small number or only one electron
- Interaction between the spins on two neighboring dots $\sim JS_1 \cdot S_2$, $S_i = (S_{ix}, S_{iy}, S_{iz})$ are spin operators
- This is called exchange interaction and can be controlled e.g. by a magnetic field
- Since it is difficult to control the magnetic field on micrometer scales, a combination of external magnetic field and electric gating is advantageous

Quantum dot quantum processor



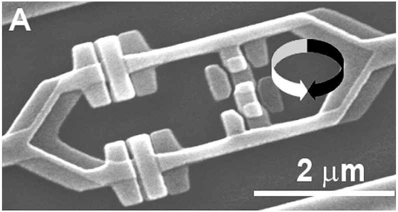
- Loss and DiVincenzo 1998
- $H = \sum_{\langle i,j \rangle} J_{ij} S_i \cdot S_j + \mu_B \sum_i g_i(t) B_i(t) \cdot S_i$
- Experimental status: coupled quantum dots with single electrons and basic single qubit gates have been demonstrated

Superconducting qubits

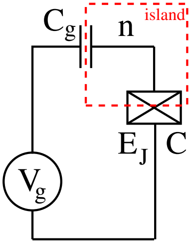
- Superconductors are macroscopic quantum systems that show infinite conductivity at temperatures below a critical temperature
- They are characterized by a macroscopic wavefunction $\Psi = \sqrt{n_s} e^{i\varphi}$
- The macroscopic supercurrent is proportional to $\nabla\varphi$
- Microscopic picture: electrons form Cooper pairs
- Two superconductors separated by an oxide barrier form a tunnel junction or Josephson junction
- The supercurrent flowing through the junction is proportional to $\sin(\varphi)$, where $\varphi = \varphi_2 - \varphi_1$ and φ_1, φ_2 are the phases on the left and right side of the tunnel junction
- This phenomenon is called Josephson effect
- The Hamiltonian leading to this current is $H = -E_J \cos\varphi$

Superconducting qubits II

- Where is the 2-level system ?
- May be based on flux or charge degrees of freedom
- Flux qubit: superconducting loop with two accessible flux states (clockwise or counter-clockwise current), Chioresco et al., Science 2003

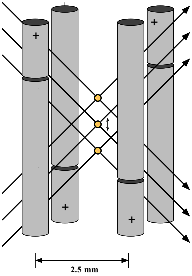


Superconducting qubits III



- Charge qubit
- Superconducting island with two accessible charge states (0 or 1 excess Cooper pairs)

Trapped ions



- N ions (e.g., ^{9}Be, ^{40}Ca) trapped in an electromagnetic trap
- each ion has two (meta-)stable states, energy separation $\hbar\omega_0$
- Laser beams induces transitions between the two states (1-qubit operation)
- Ions repel each other $\Rightarrow$ phonon-like modes along the chain, useful for coupling many qubits